IMPLICATIONS OF NEWMAN RESEARCH FOR THE ISSUE OF "WHAT IS BASIC IN SCHOOL MATHEMATICS?"

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In many countries there is a strongly held public view that the most important goal of elementary school mathematics is for young children to acquire a working knowledge of the four operations and, in particular, to be able to obtain correct answers to pencil-and-paper number questions that require the application of standard algorithmic procedures (such as vertical addition and subtraction, long multiplication, short and long division).

The main purpose of this paper is to challenge this view by summarising data from Newman error analysis studies carried out in several countries which suggest that society in general, and teachers of mathematics and mathematics teacher educators in particular, urgently need to revise the traditional view of what constitutes "basic skills" in mathematics education.

The Newman data indicate that an understanding of the vocabulary and the semantics of elementary school mathematics is fundamentally important. What is the point of children being able to carry out the mechanics of standard algorithms for the four arithmetic operations if, given a mathematics problem (verbally or in writing), they either cannot understand the problem, or they cannot work out an appropriate sequence of operations?

THE NEWMAN HIERARCHY OF ERROR CAUSES FOR WRITTEN MATHEMATICAL TASKS

Since 1977 when Newman (1977a,b) first published data based on a system she had developed for analysing errors made on written tasks, a steady stream of research papers reporting studies from many countries has appeared, in which her data collection and data analysis methods have been used (see, for example, Casey, 1978; Clarkson, 1980, 1983, 1991; Clements, 1980, 1982; Marinas and Clements, 1990; Watson, 1980).

The findings of these studies have been sufficiently different from those produced by other error analysis procedures (for example, Hollander, 1978; Lankford, 1974; Radatz, 1979), to attract considerable attention from both the international body of mathematics education researchers (see, for example, Dickson, Brown and Gibson, 1984; Mellin-Olsen, 1987; Zepp, 1989) and teachers of mathematics. In particular, analyses of data based on the Newman procedure have drawn special attention to (a) the influence of language factors on mathematics learning; and (b) the inappropriateness of many "remedial" mathematics programs in schools in which there is an over-emphasis on the revision of standard algorithms (Clarke, 1989).

The Newman Procedure

According to Newman (1977a,b; 1983), a person wishing to obtain a correct solution to an arithmetic word problem such as "The marked price of a book was $20. However, at a sale, 20% discount was given. How much discount was this?", must ultimately proceed according to the following hierarchy:

1. Read the problem;

2. Comprehend what is read;

3. Carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy;

4. Apply the process skills demanded by the selected strategy; and

5. Encode the answer in an acceptable written form.

Newman used the word "hierarchy" because she reasoned that failure at any level of the above sequence prevents problem solvers from obtaining satisfactory solutions (unless by chance they arrive at correct solutions by faulty reasoning).

Of course, as Casey (1978) pointed out, problem solvers often return to lower stages of the hierarchy when attempting to solve problems, especially those of a multi-step variety. (For example, in the middle of a complicated calculation someone might decide to reread the question to check whether all relevant information has been taken into account.) However, even if some of the steps are revisited during the problem-solving process, the Newman hierarchy provides a fundamental framework for the sequencing of essential steps.

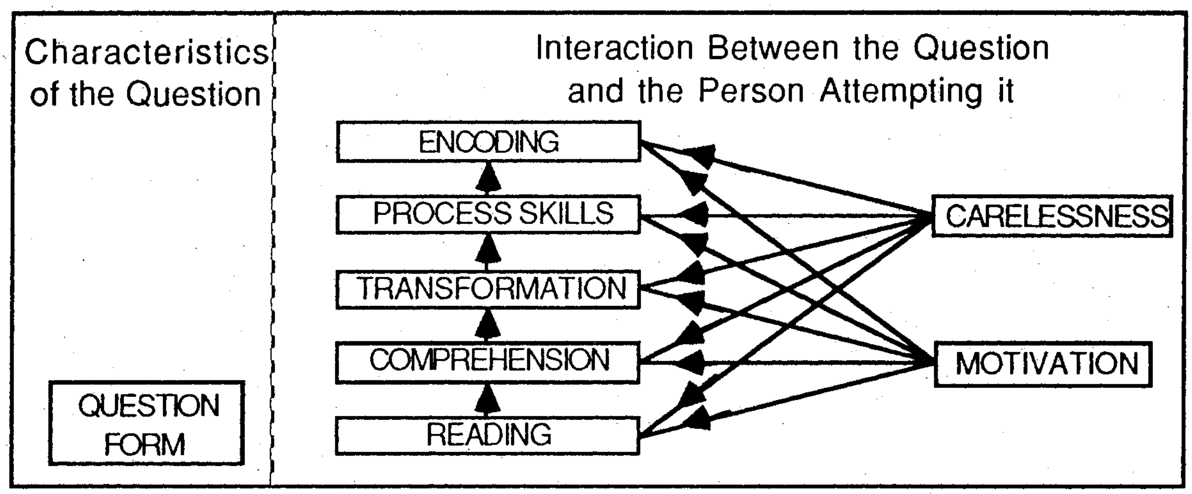

Figure 1: The Newman hierarchy of error causes (from Clements, 1980, p. 4).

Clements (1980) illustrated the Newman technique with the diagram shown in Figure 1. According to Clements (1980, p. 4), errors due to the form of the question are essentially different from those in the other categories shown in Figure 1 because the source of difficulty resides fundamentally in the question itself rather than in the interaction between the problem solver and the question. This distinction is represented in Figure 1 by the category labelled "Question Form" being placed beside the five-stage hierarchy. Two other categories, *Carelessness* and *Motivation*, have also been shown as separate from the hierarchy although, as indicated, these types of errors can occur at any stage of the problem-solving process. A careless error, for example, could be a reading error, a comprehension error, and so on. Similarly, someone who had read, comprehended and worked out an appropriate strategy for solving a problem might decline to proceed further in the hierarchy because of a lack of motivation. (For example, a problem-solver might exclaim: "What a trivial problem. It's not worth going any further.")

Newman (1983, p. 11) recommended that the following "questions" or requests be used in interviews carried out in order to classify students' errors on written mathematical tasks:

1. Please read the question to me. *(Reading)*

2. Tell me what the question is asking you to do. *(Comprehension)*

3. Tell me a method you can use to find and answer to the question. *(Transformation)*

4. Show me how you worked out the answer to the question. Explain to me what you are doing as you do it. *(Process Skills)*

5. Now write down your answer to the question. *(Encoding)*

If pupils who originally gave an incorrect answer to a word problem gave a correct answer when asked by an interviewer to do it once again, the interviewer should still make the five requests in order to investigate whether the original error was due to carelessness or motivational factors.

Example of a Newman Interview

Mellin-Olsen (1987, p. 150) suggested that although the Newman hierarchy was helpful for the teacher, it could conflict with an educator's aspiration "that the learner ought to experience her own capability by developing her own methods and ways." We would maintain that there is no conflict as the Newman hierarchy is not a learning hierarchy in the strict Gagné (1967) sense of that expression. Newman's framework for the analysis of errors was not put forward as a rigid information processing model of problem solving. The framework was meant to complement rather than to challenge descriptions of problem-solving processes such as those offered by Polya (1973). With the Newman approach the researcher is attempting to stand back and observe an individual's problem-solving efforts from a coordinated perspective; Polya (1973) on the other hand, was most interested in elaborating the richness of what Newman termed *Comprehension* and *Transformation*.

The versatility of the Newman procedure can be seen in the following interview reported by Ferrer (1991). The student interviewed was an 11-year-old Malaysian primary school girl who had given the response "All" to the question "My brother and I ate a pizza today. I ate only one quarter of the pizza, but my brother ate two-thirds. How much of the pizza did we eat?" After the student had read the question correctly to the interviewer, the following dialogue took place. ("I" stands for Interviewer, and "S" for Student.)

I: What is the question asking you to do?
S: Uhmm . . . It's asking you how many . . . how much of the pizza we ate in total?
I: Alright. How did you work that out?
S: By drawing a pizza out ... and by drawing a quarter of it and then make a two-thirds.
I: What sort of sum is it?
S: A problem sum!

I: Is it adding or subtracting or multiplying or dividing?
S: Adding.
I: Could you show me how you worked it out? You said you did a diagram. Could you show me how you did it and what the diagram was?
S: (Draws the diagram in Figure 2A.) I ate one-quarter of the pizza (draws a quarter*).

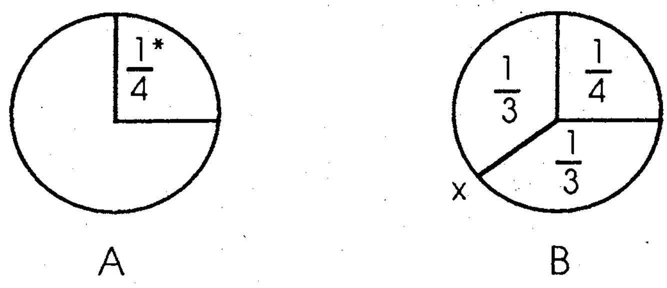

Figure 2: Diagrammatic representations of the pizza problem.

I: Which is the quarter?
S: This one. (Points to the appropriate region and labels it 1/4.)
I: How do you know that's a quarter?
S: Because it's one-fourth of the pizza. Then I drew up two-thirds, which my brother ate. (Draws line *x* - see Figure 2B - and labels each part 1/3)
I: And that's 1/3 and that's 1/3. How do you know it's 1/3.
S: Because it's a third of a pizza.

(From Ferrer, 1991, p. 2)

The interview continued beyond this point, but it was clear from what had been said that the original error should be classified as a *Transformation* error - the student comprehended the question, but did not succeed in developing an appropriate strategy. Although the interview was conducted according to the Newman procedure, some of the student's difficulties were identified without forcing her along a solution path she had not chosen.

DATA FROM STUDIES BASED ON THE NEWMAN APPROACH

Data From Early Australian Studies

In her initial study, Newman (1977a) found that *Reading, Comprehension*, and *Transformation* errors made by 124 low-achieving Grade 6 pupils accounted for 13%, 22% and 12% respectively of all errors made. Thus, almost half the errors made occurred before the application of process skills. Studies carried out with primary and junior secondary school children in Melbourne, Australia, by Casey (1978), Clements (1980), Watson (1980), and Clarkson (1980) obtained similar results, with about 50% of errors first occurring at the *Reading*, *Comprehension* or *Transformation* stages. Casey's study involved 116 Grade 7 students, Clements's sample included over 700 children in Grades 5 to 7, Watson's study was confined to a preparatory grade, and Clarkson's sample contained 13 low-achieving Grade 7 students. In each study all students were individually interviewed and with the exception of Casey, who helped interviewees over early break-down points to see if they were then able to proceed towards satisfactory solutions, error classification was based on the first break-down point on the Newman hierarchy.

The consistency of the findings of these Melbourne studies involving primary and junior secondary students contrasted with another finding, also from Melbourne data, by Clarkson (1980) that only about 15% of initial errors made by 10th and 11th Grade students occurred at any one of the *Reading*, *Comprehension* or *Transformation* stages. This contrast raised the question of whether the application of the Newman procedure at different grade levels, and in different cultural contexts, would produce different error profiles.

Newman Data From Studies in Asia and Papua New Guinea

Since the early 1980s the Newman approach to error analysis has increasingly been used outside Australia. Clements (1982) and Clarkson (1983) used the approach in error analysis research carried out in Papua New Guinea, and more recently the methods have been applied to mathematics and science education research studies in Brunei (Mohidin, 1991), India (Kaushil, Sajjin Singh and Clements, 1985), Indonesia (Ora, 1992), Malaysia (Clements and Ellerton, 1992; Kownan, 1992; Marinas and Clements, 1990; Teoh, 1991), Papua New Guinea (Clarkson, 1991), the Philippines (Jimenez, 1992), and Thailand (Singhatat, 1991; Sobhachit, 1991).

Rather than attempt to summarise the data from all of these Asian studies, the results of five studies which focused on errors made by children on written mathematical tasks will be given special attention here. The five studies, which have been selected as typical of Newman studies conducted outside Australia, are those by Clarkson (1983), Kaushil et al. (1985), Marinas and Clements (1990), Singhatat (1991), and Clements & Ellerton (1992). Pertinent features of these studies, conducted in Papua New Guinea (PNG), India, Malaysia, Thailand, and Malaysia respectively, have been summarised in Table 1.

The percentage of errors classified in each of the major Newman categories in these five studies is shown in Table 2. The last column of this table shows the percentage of errors in the categories when the data from the five studies are combined.

From Table 2 it can be seen that, in each of the studies, over 50% of the initial errors made were in one of the *Reading*, *Comprehension*, and *Transformation* categories. The right-hand column of Table 2 shows that over 60% of students' initial breakdown points in the five studies were in one of the *Reading*, *Comprehension*, and *Transformation* categories. This means that, for most errors, students had either not been able to understand the word problems or, if understanding had been present, they had not devised appropriate strategies for solving the given problems.

Table 1: *Background Details of the Asian and PNG Studies*

Study	Country	Grade level	Sample size	Number of errors analysed	Language of test & of Newman interview	Was interview in student's language of instruction?
Clarkson (1983)	*PNG*	6	95	1851	English	Yes
Kaushil et al. (1985)	*India*	5	23	327	English	Yes
Marinas & Clements (1990)	*Malaysia*	7	18	382	Bahasa Malaysia	Yes
Singhatat (1991)	*Thailand*	9	72	220*	Thai	Yes
Clements & Ellerton (1992)	*Malaysia*	5	44	497	Bahasa Malaysia	Yes

* Note that the 38 errors attributed by Singhatat to "lack of motivation" have not been taken into account for the purposes of this Table.

Table 2: *Percentage of Initial Errors in Different Newman Categories in the Four Studies*

Error Type / Study	Clarkson (1983) (1851 errors) %	Kaushil et al (1985) (329 errors) %	Marinas & Clements (1990) (382 errors) %	Singhatat (1991) (220 errors) %	Clements & Ellerton (1992) (497 errors) %	Overall %
Reading	12	0	0	0	2	7
Comprehension	21	24	45	60	26	28
Transformation	23	35	26	8	40	26
Process Skills	31	16	8	15	7	22
Encoding	1	6	0	0	6	2
Careless	12	18	21	16	20	15

DISCUSSION

The high proportion of *Comprehension* and *Transformation* errors in Table 2 suggests that many Asian and Papua New Guinea children have considerable difficulty in understanding and developing appropriate representations of word problems. This raises the question of whether *too much* emphasis is placed in their schools on basic arithmetic skills, and not enough on the peculiarities of the language of mathematics.

Table 3: *Percentage of Indian and Australian Grade 5 Children Correct on Selected Problems. (from Kaushil et al., 1985).*

Question	% Indian sample correct	% Australian sample correct
940 - 586 = q	96	75
273 ÷ 7 = q	76	55
A shop is open from 1 pm to 4 pm. For how many hours is it open?	44	87
It is now 5 o'clock. What time was it 3 hours ago?	47	88
Suniti has 3 less shells than Aarthi. If Suniti has 5 shells, how many shells does Aarthi have?	42	73

Further evidence for a possible over-emphasis on algorithmic skills was obtained in the Indian study (Kaushil et al., 1985) when the performances of the Delhi Grade 5 sample on a range of mathematical problems where compared with those of Australian fifth-grade children on the same problems. It was found that the Indian children consistently and significantly outperformed a large sample of Australian children on tasks requiring straightforward applications of algorithms for the four arithmetic operations (for example, 940 - 586 = q). However, on word problems, the Australian children invariably performed significantly better (see Table 3). Clements and Lean (1981) reported similar patterns when the performances of Papua New Guinea and Australian primary school students were compared on tasks similar to those shown in Table 3.

Interestingly, Faulkner (1992), who used Newman techniques in research investigating the errors made by nurses undergoing a calculation audit, also found that the majority of errors the nurses made were of the *Comprehension* or *Transformation* type.

SOME CONCLUDING COMMENTS

The high percentage of *Comprehension* and *Transformation* errors found in studies using the Newman procedure in the widely differing contexts in which the above studies took place has provided strong evidence for the fundamental importance of language factors in the development of mathematical concepts.

In many countries there is a strongly held public view that the most important goal of elementary school mathematics is for young children to acquire a working knowledge of the four number operations and, in particular, to be able to obtain correct answers to pencil-and-paper number questions that require the application of standard algorithmic procedures (such as vertical addition and subtraction, long multiplication and short and long division). The main purpose of this paper has been to challenge this view by summarising data from Newman error analysis studies carried out in several countries that suggest that society in general, and teachers of mathematics and mathematics teacher educators in particular, urgently need to revise the traditional view of what constitutes "basic skills" in mathematics education. Clearly, language factors should be incorporated into the public definition of what constitutes "basic skills" in school mathematics.

The Newman data indicate that an understanding of the vocabulary and the semantics of elementary school mathematics should be a central feature of mathematics curricula. What is the point of children being able to carry out the mechanics of standard algorithms for the four arithmetic operations if, given a mathematics problem (verbally or in writing), they either cannot understand the meaning of the problem or they cannot work out the appropriate sequence of operations?

The research reported in this paper raises the difficult issue of what educators can do to improve a learner's comprehension of mathematical text or ability to transform, that is to say, to identify an appropriate way to assist learners to construct sequences of operations that will solve a given word problem. At present, little progress has been made on this issue, and it should be an important focus of the mathematics education research agenda during the 1990s.

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